On Fine's Fragmentalism

Fragmentalism is the view that reality is not a metaphysically unified place, but fragmented in a certain sense, and constituted by incompatible facts across such fragments. It was introduced by Kit Fine in a discussion of tense realist theories of time (2005). Here I discuss the conceptual foundations of fragmentalism, identify several open questions in Fine's characterization of the view, and propose an understanding of fragmentalism that addresses these open questions.

Introduction

Fine introduces fragmentalism in his 'Tense and Reality' (2005). His defense of the position is tentative. He is merely out to defend two conditional claims, namely that 'if one is going to be a realist about tense, then one should be a non-standard realist and that, if one is going to be a non-standard realist, then one should be a fragmentalist' (Fine 2005: 262). He argues that fragmentalism is in a better position to account for the passage of time, that it can account for the stable truth of tensed utterances, and that it is compatible with the special theory of relativity. He also argues that it is conceptually more stable than another non-standard view (called external relativism) and, finally, that the framework enables us to clarify the distinction that some philosophers draw between an empirical and metaphysical self. I find most of these claims convincing but will not discuss them here.

I will focus solely on the conceptual foundations of fragmentalism. This should be of independent interest. Fragmentalism questions the deeply ingrained presumption that the world is a unified place, and that it cannot include incompatible facts. If such a view is conceptually stable and intelligible, this shakes a basic assumption that informs much of our theorizing about the world.

The plan is as follows: §1 offers an exposition of Fine's understanding of fragmentalism, §2 offers a critical discussion that points out open questions that need to be addressed, and §3 proposes an understanding of fragmentalism that addresses these open questions.

1. Fine's Characterization of Fragmentalism

Let us start with Fine's original characterization of fragmentalism. An important methodological starting point of Fine's approach is that we sharply distinguish between stating *what is the case* and stating *what is the case in reality* (2005: §2; see also his 2001: §8-10). We come to the world employing all sorts of representations of it, but we arguably also come with a concept of reality (or of the world as it is in itself) and we do not typically take all aspects of all our

representations to reflect what we take reality to be like. There is everything we ordinarily take to be true, and then there are the things we take to be true *and also* take to reflect what reality is like. Fine proposes that we make this distinction explicit in terms of a sentential operator '$\mathfrak{R}$' ('it is the case in reality that…'). A sentence of the form '$\mathfrak{R}A$', says that it is the case in reality that *A*.

It can be difficult to discuss matters in terms of these reality statements however and so, for ease of expression, Fine proposes that we also speak more loosely and talk of facts that belong, constitute or compose reality (2005: 268). It is important to keep in mind however that this is only loose talk, and that, on the official view, we do not refer to facts or to reality. They are not in the ontology of fragmentalism.

Fine discusses how various views of time employ different concepts of reality. He formulates four general metaphysical principles concerning reality whose conflict lie at the heart of McTaggart's argument for the unreality of time, and the resolution of which results in different theories of time (2005: 271, 280-283):[1]

> *Realism*: Reality is constituted (at least, in part) by tensed facts. In official idiom: it is the case that $\mathfrak{R}A$, where '*A*' is a tensed sentence. For example: it is the case that $\mathfrak{R}$(KF is sitting).
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> *Neutrality*: No time is privileged, the tensed facts that constitute reality are not oriented towards one time as opposed to another. In official idiom: it is the case that $\mathfrak{R}A$ and $\mathfrak{R}B$ where '*A*' and '*B*' are descriptions of how reality is at different times. For example: it is the case that $\mathfrak{R}$(Aristotle is sitting) and $\mathfrak{R}$(KF is sitting).
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> *Absolutism*: The constitution of reality is an absolute matter, i.e. not relative to a time or other form of temporal standpoint. In official idiom: it is not the case relative to one time *t* that $\mathfrak{R}A$ and the case relative to another time *t** that $\neg\mathfrak{R}A$.
>
> *Coherence*: Reality is not contradictory, it is not constituted by facts with incompatible content. In official idiom: it is not the case that $\mathfrak{R}$A and $\mathfrak{R}$B where 'A' and 'B' state incompatible facts.. For example: it is not the case that $\mathfrak{R}$(KF is sitting) and that $\mathfrak{R}$(KF is standing).

Fragmentalism accepts the first three of these principles but denies the last of these, *Coherence*. Reality is constituted by all the tensed facts that obtain at some time or other, even though some of these are incompatible, such as *the fact that KF is sitting* and *the fact that KF is standing*.

The facts stand in a relation that Fine calls coherence. He proposes that we take this relation of coherence between facts as a primitive notion (2005: 281). Using

[1] The statements of the principles in the informal idiom are Fine's exact formulations; their translations into the official idiom are mine.

this relation, we can identify *maximal coherent collections of facts*, the 'fragments'. Each moment of time is identified with such a fragment. This means that *the fact that KF is sitting* coheres at least with all the facts that obtain simultaneously with it, so that the collection of facts (i.e. the relevant fragment) corresponds exactly to the contents of a moment of time. Just as reality is not on the official view an entity over and above the facts that are the case in reality, so is a time not an entity over and above the facts that are the case when it is that time.

The division of reality into internally coherent fragments plays an important role in the semantics that accompanies the metaphysics (see 2005: 282, 295-298). An utterance is taken to come with an in-build specification of the fragment within which the verifying facts are to be found, a 'focal content'. An utterance is by default focused on the fragment (or time) of utterance, and the utterance will be false when the required facts are not found within the specified fragment, even though they are found in a different fragment. Because all the facts that belong to a single fragment are compatible, and because the utterance is focused on a single fragment only and not on the fragmented reality in its totality, no utterance of a contradiction comes out true. So the semantic machinery is devised in such a way that the metaphysical picture does not legitimate contradictory utterances.

In a nutshell, then, Fine's fragmentalism pictures reality as a series of 'collections of tensed facts', each of which is internally coherent but fails to cohere with other collections of tensed facts. This does not vindicate contradictory utterances however because it is assumed that our situated language-use only ever concerns the 'collection of facts' that it is focused on and not reality at large.

2. Critical Discussion

With Fine's characterization of fragmentalism in place, I want to discuss the view in more depth. The critical discussion revolves around two questions: first the question of the exact sense in which reality is incoherent according to Fine's fragmentalism, and second the question of exactly how we are to understand the coherence relation that structures the fragmented reality.

Let me start with the first question. Fine remarks that 'the fragmentalist takes reality to be contradictory' and that she denies that 'all contradictions can be ironed out' (2005: 282, 280). This sounds worrying of course, and will be enough to deter many from the view. But it turns out that reality is contradictory in a particular sense according to Fine's fragmentalism.

We typically think of contradictions as conjunctions of contradictory conjuncts. So one might wonder first of all whether conjunctions of contradictory conjuncts are ever the case in reality, that is, whether the fragmentalist accepts that $\Re(A\&\neg A)$. Fine will reject this, given his view that 'one might want to explain the obtaining of a conjunctive fact in terms of the obtaining of its conjuncts' (2005: 281). The relevant sense of explaining here is presumably that of grounding (as understood in his 2001 and 2012): the obtaining of a conjunction is grounded in the obtaining of each of the conjuncts, and hence conjunctions are not the case in

reality. If conjunctions are not the case in reality then, ipso facto, neither are conjunctions of contradictory conjuncts the case in reality. So reality will not be contradictory in this sense, according to Fine.

But then, in what sense is it incoherent exactly? Consider the following two distinct ways in which reality may be contradictory:

I. It is the case that $\mathfrak{R}A$ and $\neg\mathfrak{R}A$, i.e. it both is and is not the case in reality that *A*.

II. It is the case that $\mathfrak{R}A$ and $\mathfrak{R}\neg A$, i.e. it is the case in reality that *A* and it is the case in reality that not *A*.

These different options come with different commitments on the part of fragmentalism.

According to option I, reality is contradictory in the sense that $\mathfrak{R}A$ and that $\neg\mathfrak{R}A$, which is to say, the statement '$\mathfrak{R}A$' is both true and not true, or the fact that *A* both belongs and doesn't belong to reality. If the fragmentalist allows such contradictions, she could not plausibly assume that classical logic is the right logic given that classical logic is explosive: a contradiction implies any arbitrary sentence. So either fragmentalism is committed to a rejection of classical logic, or this is not the way reality is incoherent according to fragmentalism.

According to option II, reality is contradictory in the sense that $\mathfrak{R}A$ and $\mathfrak{R}\neg A$. In other words, the fact that *A* and the fact that not *A* are both the case in reality. Note that if this conception is not to imply option I, we need to reject certain inferences involving the notion of reality. First we must reject that if $\mathfrak{R}\neg A$, then $\neg\mathfrak{R}A$. So we must assume that, if it is the case in reality that *not A*, this does mean that it is not the case in reality that *A*. And second, if we allow that $\mathfrak{R}A$ and that $\mathfrak{R}\neg A$, we must deny the factivity of reality, that is, we must deny that if $\mathfrak{R}A$, then *A*. As Merlo notes in a discussion of Fine's view, if the notion of reality is factive, then 'incoherence is fated to spread from reality (the totality of what is really the case) to the world (the totality of what is the case)' (2013: 6). Indeed, if we have it that $\mathfrak{R}A$ and that $\mathfrak{R}\neg A$, and that reality is factivity, this implies respectively that *A* and that $\neg A$, and hence that classical logic must again be revised, on pain of triviality.

So we need to reject the factivity of reality in order to quarantine incoherence, and we need to reject that the presence of a negative fact signifies the absence of the negated fact (or again, supplement the view with its own conception of the right logic). Concerning the factivity of reality, Fine argues that the fragmentalist should indeed deny that 'whatever facts belong to reality obtain' (2005: 297-298), i.e. should indeed deny the factivity of reality. This is what allows the fragmentalist to withstand an objection from the truth of token utterances that plagues standard tense realism (see 2005: §8).

When we deny factivity however, we lose a natural understanding of what reality consists in. Indeed, when Fine first glosses the notion of reality he remarks that 'whatever is really the case (belongs to metaphysical reality) may, with some plausibility, be taken to be the case (belong to mere reality)' (2005: 267; noted in

Merlo 2011: 5). To be sure, when providing this gloss, Fine is only introducing the generic notion of reality, which becomes refined in different ways by the different views that he discusses. The initial gloss is not presented as a core feature shared by all the different views. But it nevertheless highlights that factivity is part of the intuitive understanding of reality. Your conception of reality is plausibly reflected in a privileged subset of what you take to obtain, namely the subset of those facts that are not mere truths but that, you think, also characterize what the world is like 'in itself', i.e. what reality is like. If we reject factivity, this picture of things is not right. Some facts belong to reality without being the case at all, and what belongs to reality is not a subset of what obtains. Or, to put it informally, some of the things that belong to reality are not facts. This does not seem a very plausible notion of reality to me.

Fine states explicitly that fragmentalism takes reality to be 'irredeemably incoherent' (2005: 281) and unsurprisingly, fragmentalism has been resisted in the literature on the grounds that it offers an incoherent conception of reality (see, e.g., Deng 2013: 10-11). But before we can critically assess fragmentalism on this basis, we need to be clear on the precise sense in which it is 'incoherent'. It appears that the less problematic the contradictions that are accepted, the more problematic the conception of reality that the fragmentalist requires.

The question of the exact sense in which reality is incoherent is connected with the question of how we are to understand the coherence relation that structures the facts into fragments. The notion of 'coherence' plays a central role. There are however some open questions concerning the notion.

Note first of all that – though the relation is called 'coherence' by Fine – it must be quite different from the ordinary relation of coherence that holds amongst propositions. This is shown by Correia and Rosenkranz (2012: 312). Dubbing Fine's notion of coherence 'coherence*' to distinguish it from the ordinary understanding, they note the following:

> [S]upposing, say, that there is no time, past, present or future, at which Socrates is furious and Plato is anxious, the fragmentalist should say that there is no fragment which comprises both the fact that Socrates is furious and the fact that Plato is anxious, and accordingly that these two facts do not cohere*; and yet, these two facts cohere, in so far as it could have been the case that, at some time, Socrates is furious and Plato is anxious. Coherence* entails coherence, but the converse does not hold. (Correia and Rosenkranz 2012: 312).

If Fine's notion of coherence* were like the ordinary notion of coherence, then the reduction of times to fragments would be inadequate due to an over-generation of fragments. We would find more times than there actually are.

So the fragmentalist's notion of coherence is not charitably taken to be the ordinary notion of coherence. But if we cannot identify the notion of coherence* with a primitive that we are already familiar with, then the fragmentalist incurs the burden of elucidating it. On this, Fine writes:

> One might want to take the notion of coherence as fundamental in addition to the notion of reality. One would then expect there to be various substantive 'rules of

> coherence' concerning the conditions under which a set of facts would be coherent and the way in which the coherence of one set of facts might constrain the coherence of another set. For example, in the classic tense-logical case, one would want that if *f* were to cohere (i.e. to be simultaneous) with *g* and *g* were to cohere with *h* then *f* would cohere with *h* (though, within a relativistic setting, this rule would have to be dropped). (Fine 2005: 281-282).

So coherence is a fundamental notion that is to be elucidated in terms of rules that govern it.[2] The example rule is that the relevant notion of coherence is transitive when we ignore considerations from the special theory of relativity.

The transitivity of coherence requires that fragments do not overlap. However, intuitively, they *do* overlap. Imagine that KF and ML are both sitting, and then ML stands up while KF continues to sit. If *the fact that KF is sitting* coheres with *the fact that ML is sitting* (given the earlier situation), but also coheres with *the fact that ML is standing* (given the later situation), then the fragments would overlap and the transitivity of coherence would dictate that *the fact that ML is sitting* coheres with *the fact that ML is standing*, which certainly do not cohere. So if we indeed assume transitivity, the fragments should not overlap in this way.[3]

We can avoid the overlap if we assume a suitable view of the identity of facts and allow distinct token facts of the same type, so that each fragment consists only of its own collection of momentary facts. Now Fine seems indeed to accept such token momentary facts in the guise of token events, writing that 'it might be supposed, for example, that there are […] token events that can occur only at a given time' and that 'each current token momentary event would give rise to a tensed fact, the event's occurring, which could obtain only at the current time' (2005: 318). If we apply this suggestion in the above example, there is the first token event e_1 of KF's sitting, and there is the second token even e_2 of KF's sitting, and only the first coheres with the (token) event of ML's sitting, and only the second coheres with the (token) event of ML's standing. In terms of the official idiom, we now say that $\mathfrak{R}$(event e_1, of KF's sitting, occurs) and that $\mathfrak{R}$(event e_2, of KF's sitting, occurs), where e_1 and e_2 are distinct events.

On this view, fragmentalism comes with a commitment to token events. But such time-specific entities do not seem to be in the spirit of tense-realism, after all, we distinguish the different token events first and foremost by virtue of the fact that one occurs at one time and the other at a different time. In fact, depending on one's view of events, one might want to say that the event e_1, of KF's sitting, occurs *in virtue of* or *because* KF sits at *t*, where *t* is the time that the e_1 is specific to.[4] Instead

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[2] One might have thought that fragmentalism is simply the view that denies that if $\mathfrak{R}A$ and $\mathfrak{R}B$, then $\mathfrak{R}(A, B)$, so that our overall conception of reality would take this form: $\mathfrak{R}(A, B, \ldots)$, $\mathfrak{R}(A, \neg B, \ldots)$, … We do not then need a separate notion of coherence. This is not Fine's view, in fact, in a footnote, he suggests the opposite direction of reduction, that 'given a primitive relation of coherence, one might take a fact to belong to reality when it is self-coherent' (2005: 281, fn. 13).

[3] When Fine first introduces fragmentalism, he writes that 'reality will divide up into a number of different but possibly overlapping fragments' (2005: 281). The mentioned possibility of overlap is however being allowed by Fine in relativistic cases only, which are ignored here.

[4] Think of Quine's (1960: 171) views that events are spatiotemporal regions; Lewis' (1986) view that events are properties *of* spatiotemporal regions; Kim's (1993) view that events are compounds

of saying that $\mathfrak{R}$(event e_1, of KF's sitting, occurs) we would then say that $\mathfrak{R}$(KF is sitting at *t*), but the latter is arguably not a tensed fact. So the fragmentalist not only incurs a commitment to token events, she also requires a particular view of events that is somehow congenial to her realism about tense.

Independent of the commitment to events, I find it more natural to think that it is not all flux, with new token entities popping into existence at every moment in time, but that there is also stability in the continued obtaining of certain facts. If there is indeed such stability across time, then – contra Fine – we should rather deny that coherence is transitive, even in non-relativistic cases.

Another open question is the way in which the notion of coherence bears on our understanding of what it is for facts to be incompatible. The central thesis of fragmentalism is that reality can be constituted by incompatible facts. But what is it for facts to be incompatible? If we think that facts are incompatible if and only if they cannot both obtain in reality, then one will be unable to make sense of a view that says that *incompatible* facts *can* both obtain in reality. That is, if we plug this definition of incompatibility into the formulation of the rejected *Coherence* principle, fragmentalism becomes the view that pairs of facts that cannot belong to reality belong to reality. So if we cling to the standard understanding of incompatibility, then fragmentalism risks being outright unintelligible.

The fragmentalist must offer some understanding of incompatible facts that allows us to see how two incompatible facts can both belong to reality. The most promising proposal is that facts are incompatible if and only if it is metaphysically impossible that they cohere. We must understand coherence in such a way that the compatibility of facts can be understood as the possibility of such coherence. But to the extent we do not know what exactly can cohere with what, we do not know whether this is an adequate understanding of the compatibility and incompatibility of facts.

To be sure, the point is not that we demand an exhaustive definition of a notion that Fine wants to introduce as a primitive. The complaint is just that, in terms of elucidation, both the name that Fine picks for the notion (namely 'coherence') as well as the single feature attributed to it (namely transitivity) seem to point us in the wrong direction – and this threatens to undermine our overall understanding of fragmentalism given that the incompatibility of facts must itself be understood in terms of the notion of coherence.

3. Fragmentalism Redux

These issues show that there is work to be done. I want to take up some of that work in this final section. I will propose a way of understanding fragmentalism that I find plausible, and that answers the various open questions in a unified manner.

<*x*, F, *t*> of an object *x*, property F, and time *t*; or Chisholm's (1976: 126) view that events are states of affairs that are concretized at a place and time. For a tense realist critique of irreducible reference to events, see Prior (1968).

Fine's characterization of fragmentalism exemplifies his meta-theoretic view on how best to formulate tense realism. I believe the fragmentalist view is independent of these meta-metaphysical commitments and so I will not resort to the reality operator in stating the view, but focus on the addition of a certain concept that one may take to feature in what is the case as such, and not just in what is the case in reality. I will return to this below.

So let us ask again: what could it be for a world to be fragmented? If the world is fragmented, then *the fact that ML is sitting* obtains and that *the fact that ML is standing* obtains, but – though each fact obtains – they do not, in a certain sense, *co-obtain*. That is the central claim of the fragmentalist view that I want to propose. The relevant sense of co-obtainment will of course play the role of Fine's notion of coherence.

As in the case of Fine's notion of coherence, co-obtainment is proposed as a primitive notion, and hence we incur the burden of elucidating it. It is a tricky question what the standards are for such elucidation and what sorts of things need to be involved in this. Rosen describes a successful process of elucidation as follows (concerning the elucidation of metaphysical necessity in his case):

> If we do not begin with a definition, we must offer some sort of informal elucidation. We all know roughly how this works in other parts of philosophy. The neophyte is presented with a battery of paradigms and foils, ordinary language paraphrases (with commentary), and bits and pieces of the inferential role of the target notion, and then somehow as a result of this barrage he cottons on. (Rosen 2006: 14).

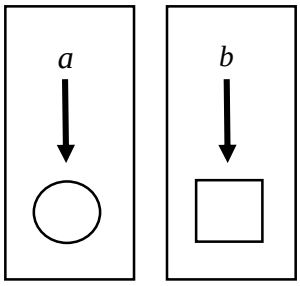
Let us employ all these means of clarification: paradigm cases, ordinary language paraphrases, models and metaphors as well as a specification of the inferential role, to elucidate the notion of co-obtainment.

Though we have introduced the notion as the co-obtainment of facts, reference to facts should be inessential and not part of the official understanding of things. To state matters directly, I will use a sentential operator '○' to form sentences of the form '*A*○*B*', which we read as '*A insofar as B*'. For example, 'Sophie is sitting ○ the tree is leafless' says that Sophie is sitting *insofar as* the tree is leafless. Needless to say, the 'insofar as' locution is used here as a theoretical phrase and doesn't necessarily share all features with its more common usage. In particular, it is not to be read in an explanatory sense, as it sometimes is, but only in a bare conjunctive sense, merely indicating that the one fact co-obtains with the other, that they obtain *together*. I will call sentences of the form '*A*○*B*' co-obtainments, in much the same way we speak of conjunctions and disjunctions, and will continue to occasionally refer to facts in the understanding that this is mere loose talk.

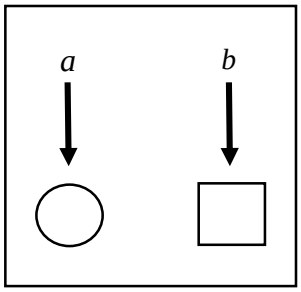
How could we paraphrase the claim that, for example, KF is standing *insofar as* ML is sitting? When two facts co-obtain, we might say that they form a unified qualitative manifestation of the relevant objects, one single bit of world within which the things are a certain way. This contrasts with the way we now understand the truth of a conjunction, which just informs us about what obtains and not about whether the one also obtains *insofar as* the other does. If we were to

'picture' the truth conditions of the conjunctive and co-obtainment claims, the latter would express a type of unity not expressed by the former (where *a* and *b* name distinct objects and the arrows represent the instantiation of the depicted shape or shade):

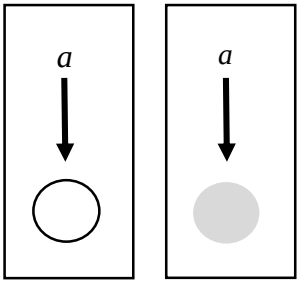
'*a* is round & *b* is square & ¬(*a* is round ∘ *b* is square)':



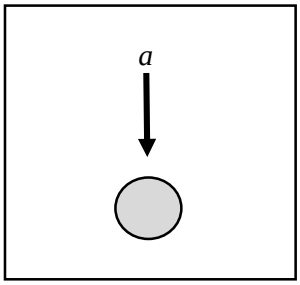
'*a* is round ○ *b* is square':



'*a* is round & *a* is coloured & ¬(*a* is round ∘ *a* is coloured)':



'*a* is round ○ *a* is coloured':



Whenever two facts fail to co-obtain, i.e. when ¬(*A*○*B*), this means that relative to the one fact, the other fact is not there at all, i.e. it means that *A* obtains *insofar as B* does not. That does not mean that *A* and *B* are not each of them the case, it only means that they are not the case *together*, that they do not make for a unified chunk of world.

Of course this talk of a 'unified qualitative manifestation' and 'forming unified chunks of world' are just informal paraphrases helping us to picture what sort of metaphysical structure co-obtainment latches onto. The meaning they convey should be paired together with the right inferential role. So let us turn to this. It will be helpful to specify the semantics of co-obtainments in terms of models of fragmented worlds.

Consider first two toy-models. The first toy-model is one gigantic picture that depicts everything that is the case within the world. We say that the picture represents something as being the case when it depicts it. This picture is then representative of a non-fragmentd world, the world as we ordinarily conceive it to be. Every fact is here depicted together with every other. The second toy-model is not one but a collection of gigantic pictures. This time, we say that these pictures only collectively depict all of reality: something is represented as obtaining when it is depicted in one of the pictures, and represented as not obtaining when not depicted in any of them. This second toy-model is of course representative of a 'fragmented world': it naturally represents that two facts co-obtain when there is a single picture that depicts both facts, and it represents that facts fail to co-obtain when there is no single picture that depicts both facts. The collection of pictures thus collectively depict a world in which not all facts that obtain also co-obtain. A complete description of the world does not just need to capture everything that obtains, it needs to capture which things co-obtain, and which things do not.

The toy models can be adapted to set theoretic models that allow us to pinpoint the inferential role more precisely.[5] (The characterization of the models can be skipped without loss. I only refer back to it in footnotes). Let our language be a set of atomic sentences, *p, q, r, ...* and complex sentences formed from these, of the form $\neg A$, $A\&B$, and $A \circ B$. A model M for this language is a triple $\langle W, v \rangle$, where W is a set of points and v assigns 1 or 0 to each atomic sentence relative to each point w in W. The assignment by v of 1 or 0 to each atomic sentence relative to each point is extended to an assignment for all the sentences via the following recursive clauses:

$v_w(A \circ B) = 1$ iff $v_w(A) = 1$ and $v_w(A) = 1$
$v_w(A\&B) = 1$ iff $v_w(A) = 1$ and $v_w(A) = 1$
$v_w(\neg A) = 1$ iff it is not the case that $v_w(A) = 1$

On the basis of these, truth in a model M, is defined via the following recursive clauses:

$M \Vdash p$ iff there is a w such that $v_w(p) = 1$
$M \Vdash A \circ B$ iff there is a w such that $v_w(A \circ B) = 1$
$M \Vdash A\&B$ iff $M \Vdash A$ and $M \Vdash B$.
$M \Vdash \neg A$ iff $M \nVdash A$

[5] The semantics is intimately related to the 'discussive logic' of Jaśkowski (1948/1969) - with the important difference that Jaśkowski's logic is paraconsistent, whereas the semantics below isn't. For interestingly related logics, see Rescher and Brandom (1980), Priest (2008) and, in particular, Restall (1997).

The validity of an inference, written $\Sigma \vDash A$, is defined as usual:

> An inference from Σ to A is valid, i.e. $\Sigma \vDash A$, iff, if $M \Vdash \Sigma$ then $M \Vdash A$.[6]

These models are clearly analogous to the toy-model. The points w in W represent of course the multiple pictures. It should be stressed however that neither the pictures in the toy-model, nor the points in the set-theoretic model correspond to anything in our ontology, and are solely parts of the models, not of what is modelled. Again, fragmentalism is not the view that there literally speaking are entities called fragments relative to which thing obtain, it is solely the views that certain facts co-obtain and others do not. We can use these models to clarify the coherence of fragmentalism, and elucidate the inferential role of co-obtainment. I will only explicitly refer to the models in the footnotes however.

First of all, we have it that:

$A \circ B \vDash B \circ A$
$A \circ (B \circ C) \vDash (A \circ B) \circ C$[7]

If A obtains insofar as B does, then B obtains insofar as A does. And if A co-obtains with the co-obtaining of A and B, then the co-obtaining of A and B co-obtains with C.

Co-obtainment is not transitive:

$A \circ B, B \circ C \nvDash A \circ C$[8]

If A co-obtains with B, and B with C, this doesn't mean that A co-obtains with C. This addresses the transitivity worry for Fine's fragmentalism that we discussed above, and allows us to secure the possible stability of facts across time. If ML is sitting *insofar as* KF is sitting, and then ML is standing *insofar as* KF is sitting, it does not follow that ML is standing *insofar as* ML is sitting, as it should not.

Co-obtainment also does not satisfy adjunction:

$A, B \nvDash A \circ B$[9]

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[6] By $M \Vdash \Sigma$ we mean that $M \Vdash B$ for all $B \in \Sigma$.

[7] To see why co-obtainment is associative, note that if we have $A \circ (B \circ C)$ this means, by the V-clause for co-obtainment that there is a point w where both A and $B \circ C$ are true, but by the v-clause the latter is only true at w if both B and C are true at w. This means that all three, A, B and C are true at w, which means that $A \circ B$ must be true at w together with C, and hence that $(A \circ B) \circ C$ is true in the model.

[8] Consider a model where we have a point w_1 at which atomic sentences p and q are true but r isn't, and a point w_2 at which q and r are true but p isn't. In such a model, $p \circ q$ and $q \circ r$ are true, but $p \circ r$ isn't.

[9] Consider a model where we have w_1 at which p is true and w_2 at which q is true. Here p and q are true, but $p \circ q$ isn't true, given that there is no point at which p and q are *both* true.

This is of course as it should be. The central claim of the fragmentalist is that two facts may each obtain without them co-obtaining, i.e. that it is a contingent and substantial matter whether two compatible co-obtain or fail to do so. The failure of the adjunction rule makes clear how co-obtainment is distinct from conjunction, for which these rules do hold. We cannot infer $A \circ B$ from A and B, whereas we *can* infer $A \& B$ from A and B. The claim that $A \circ B$ says something more than that A and B are each the case it says that they are the case together.

Co-obtainment also does not in general satisfy simplification:

$A \circ B \not\vDash A$[10]

Think back to the toy models. Say that there is picture on which the sun shines and a picture on which it rains. Then we say that the sun shines, and that it rains. But we also say that the sun shines *insofar as* it does not rain and that it rains *insofar as* the sun does not shine. But the claim that it does not rain *insofar as* the sun shines does not imply that it does not rain, because it does. To put it quasi-formally: *it rains* $\circ$ ¬(*the sun shines)* does not imply ¬(*the sun shines*).

Interestingly, in the special case where co-obtainment embeds an atomic claim, we do have simplification for co-obtainment with respect to that claim:

$p \circ A \vDash p$, where p is an atomic sentence.[11]

In particular, this means that there is now a sharp distinction between atomic sentences and negations of them. From the fact that KF is sitting *insofar as* it is not the case that ML is standing, we can infer that KF is sitting but we cannot infer that it is not the case that ML is standing. Imagine we view the fragmented world *sub specie aeternitatis*, and ask 'is KF standing?' The answer to this is yes he is, over here. If we also ask 'is it not the case that ML is standing?' The answer is, no, because he *is* standing, over there. Speaking informally, whenever there is time when ML is standing, it is the case that ML is standing but when there is a time when ML is not standing, it does not follow that ML is not standing because there might also be a time where he is. This asymmetric treatment of atomic claims and negations of them only reflects a natural understanding of negation. When one fact does not obtain *insofar as* another fact obtains, this does not suffice for the non-obtaining of that fact.

Not only is this the natural way to understand negative facts in this context, it also underwrites the logical coherence of the framework. In particular, contradictory sentences A and $\neg A$, are never both true. A negative claim $\neg A$ is true

[10] Consider a model where we have w_1 at which p is true and w_2 at which $\neg p$ is true. At w_2, $\neg p \circ \neg p$ is true, and hence that this co-obtainment is true simpliciter in the model. But $\neg p$ is not true in the model, given that p is true at w_1. So this is a case where we have $\neg p \circ \neg p$ but not $\neg p$.

[11] If $p \circ A$ is true, then there is a point w at which p and A are true, but by the truth-in-a-model clause for atomic sentences, this suffices for p to be true in the model.

if and only if *A* is not true. The proposed understanding of fragmentalism underwrites the law of non-contradiction:[12]

$$\vDash \neg(A\&\neg A)$$

There is a sharp distinction between incompatible facts and contrary ones. The conjunctive claim that two metaphysically incompatible matters are each the case can be true, but the conjunctive claim that a certain matter is both the case and not the case cannot be true. Though KF can be sitting and be standing in the world at large (as long as these facts do not co-obtain), he cannot both be sitting and fail to be sitting.

The fragmentalist can sensibly allow that incompatible statements are true because of the way incompatibility is naturally understood on the proposed view. When two facts co-obtain, we said that they form a unified qualitative display of the objects involved. This means that two facts are naturally taken to be incompatible when they cannot constitute such a single chunk of reality, i.e. when they cannot co-obtain:

> Incompatibility: *the fact that A* and *the fact that B* are incompatible iff_{df} necessarily $\neg(A \circ B)$.

When we try to imagine *a* as co-instantiating *roundness* and *squareness* and see that this could never work, this is precisely to imagine its being square and its being round as co-obtaining in the sense that we are after, it's being both *together at once*. It is this understanding of incompatible facts that allows us to make sense of the obtaining of two incompatible facts: though two incompatible facts cannot co-obtain, they can both obtain (without also co-obtaining).

This means that a common inference becomes fallacious within this understanding of fragmentalism. We normally assume that the truth of a sentence suffices for the falsehood of any sentences that are incompatible with it. Writing 'Inc(*A*, *B*)' to designate that two sentences state incompatible facts, it is normally assumed that if Inc(*A*, *B*) and *A*, then we can infer that ~*B*. For example, if the fact that KF is sitting is incompatible with the fact that KF is standing, then if KF is sitting we assume that this implies that KF is not standing. If the world can be fragmented however, then just because two sentences designate incompatible affairs does not mean that the truth of the one suffices for the falsehood of the other. The presumption that accepting incompatible states will lead to an incoherent conception of reality arises because we infer contrary states from

incompatible ones.. But this translation of incompatible truth conditions into incoherent truth conditions implicitly assumes what the fragmentalist denies, namely that the reality of one fact leaves no room for any facts that are incompatible with it, which would be the case if, necessarily, all facts co-obtain. But, if the world is fragmented, they do not.

[12] Indeed, given that negation is classical, we also have the principle of explosion (or *ex falso quodlibet*) $A\&\neg A \vDash B$.

Thus, we can see that the fragmentalist framework should no longer be described as offering an 'incoherent' picture of reality or resisting that all 'contradictions' can be ironed out – contrary to Fine's gloss on the view. All contradictions should be ironed but, when we do so, we may still be left with the statements of incompatible facts that do not co-obtain and so stay out of each other's hairs.

Given the proposed framework, what does our conception of a fragmented world look like more concretely? This will depend on various factors, on where we think fragmentations occur (just across time, or in other ways as well?), and on whether we think co-obtainment is involved in fundamental facts and, if so, how we think of those fundamental facts independently of fragmentalism. With regard to the latter, the fragmentalist framework is compatible with different views of fundamental reality. If we assume a view of fundamentality according to which differences in logical complexity do not make for a difference in fundamentality (as in e.g. Sider 2011: §8.3), then A and B may be just as fundamental as $A \circ B$. If we assume however a more Finean view of fundamental reality, on which logically complex are typically grounded in simpler truths and hence not part of fundamental reality (see Fine 2012: §1.6), then it seems that atomic sentences are grounded in co-obtainment statements involving them, i.e. p and q will plausibly be grounded in the fact that $p \circ q$ given that the latter suffices for the former two to obtain but not vice versa. This means that the atomic facts give way to co-obtainment facts in our conception of reality. Reality will consist solely in terms of co-obtainment facts and their negations, and we arrive at a view according to which the overall fragments are prior to the facts that constitute them, a type of fragment monism.[13]

Not also that, if the proposed framework is combined with a Finean view of fundamentality, reality can be factive, i.e. we can allow that if $\mathfrak{R}A$ then A, without this leading to any contradictions spreading from what is the case in reality to what is the case as such. As we have seen, the description of a fragmented world does not imply contradictions. Similarly, we can allow that $\mathfrak{R}\neg A$ if and only if $\neg\mathfrak{R}A$, i.e. that the presence of a negative fact signifies the absence of the negated fact from reality. So not only is the framework more neutral, it shows how the fragmentalist view can be paired with a more plausible conception of reality.

Conclusion

I do not want to claim that the proposed understanding of fragmentalism is the only way of filling out the view, or that it offers the best fit with Fine's characterizations. I do want to claim however that this is a sensible understanding of the view, which addresses all the issues we encountered in §2. We have spelled out in quite some detail what it is for facts to co-obtain, how we can understand the incompatibility of facts in terms of their inability to co-obtain, and thus how the fragmentation of

[13] Thanks to an anonymous referee for pointing this out.

reality allows for the obtaining of incompatible facts. If one is going to be a fragmentalist, then this should be the view that to adopt.[14]

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