Perspectival Variance and Worldly Fragmentation

Martin A. Lipman

Objects often manifest themselves in incompatible ways across perspectives that are epistemically on a par. The standard response to such cases is to deny that the properties things appear to have from different perspectives are properties things really have out there. This type of response seems worrying: too many properties admit of perspectival variance and there are good theoretical reasons to think that such properties are genuinely instantiated. So we have reason to explore views on which things can have the incompatible properties they appear to have across perspectives. This paper explores the view that things can have incompatible properties if the world is not a metaphysically unified place, but fragmented. There is a sensible notion of co-obtainment on which two facts can each obtain without co-obtaining. Using this notion, we can step back from our embedded perspectives on the world without deeming the contents of those perspectives to be mere appearances. This renders the sheer pervasiveness of perspectival variance a serious pressure on the standard response as well as a serious reason to think that the world is indeed fragmented.

Key words: fragmentalism, perspectival facts, secondary properties, co-obtainment, metaphysical relativism, manifest image

1. Introduction

> [B]ecause honey appears bitter to some and sweet to others, Democritus said that it is neither sweet nor bitter, and Heraclitus that it is both. [Sextus Empiricus *Outlines of Scepticism*: II.63]

We live in a Democritean age. Whenever something appears in incompatible ways across perspectives that are epistemically on a par, the standard response is to think that the incompatible ways things appear to the observers are not genuine ways things are out there. They are ways things *merely* appear to the observers in their respective circumstances.

Is there even a coherent alternative? There is, or so this paper argues. We can make sense of a notion of co-obtainment according to which two facts can both obtain yet fail to co-obtain, making for a metaphysical fragmentation of the world. In particular, it's possible that two incompatible facts both obtain as long as they do not co-obtain. I will clarify the semantics of this notion of co-obtainment, and show that the view doesn't require a paraconsistent logic.

The notion of fragmentation has been invoked to explain how a non-trivial body of information could contain inconsistencies and yet be closed under a

consequence relation (notably in [Lewis 1982] and [Stalnaker 1984: Ch.5]). And the idea that the world is itself fragmented is invoked by Fine [2005] to defend the reality of tensed facts and subjective facts. I will not discuss these precursors.[1] I will focus on the way the possible fragmentation of the world affects the way in which the world of experience (or the manifest image) feeds into our conception of what the world consists in. I will not attempt to convince my reader that the world is fragmented in some particular way, or to offer a fragmentalist theory of any particular perspectival phenomenon.

The paper has the following structure: first I discuss the issues arising from instances of perspectival variance (§1), then briefly why metaphysical relativism doesn't seem a conceptually stable response (§2), and then – in the second half of the paper – I propose a way of making sense of worldly fragmentation (§3 and §4).

2. The standard response to perspectival variance

Imagine that we face each other, and that Sophie and Nora stand between us. We observe things differently: you observe Sophie as being on the left of Nora, I observe Sophie as being on the right of Nora. Sophie and Nora are observed to have incompatible two-place relations from our different perspectives. We think that Sophie cannot be both on the left and on the right of Nora, and, given that no orientation is privileged as the one way of seeing the world as it truly is, nor is Sophie just on the left of Nora, or just on the right of her. Since there is nothing special about the case, we draw the general conclusion that there are no two-place relations of *being on the left of* or *being on the right of* instantiated by pairs of things out there in the world.

In general, say that we have a case of perspectival variance iff: (1) under one set of conditions C_1 it appears to an observer that *A*, and (2) under another set of conditions C_2 it appears to an observer that *B*, where (3) '*A*' and '*B*' state incompatible facts, and (4) the sets of condition C_1 and C_2 are epistemically on a par and not worse than the standard conditions of observing things to be as '*A*' and '*B*' describe them to be. By 'conditions of observation' I mean some fact that obtains and which is such that, if it didn't obtain, the content of the observation would be different. If I observe a blue sky, the fact that I do so under certain lighting conditions, at a certain time, using a certain visual apparatus, and even the very fact that I'm observing the blue sky, are all amongst the conditions of observation. I will also refer to the conditions of observation as 'perspectives'. I assume that the 'ways in which things appear to an observer' are ways things appear *to be*, i.e. properties things appear to have when observed (see [Shoemaker 2006: 461]). This means that sentences '*A*' and '*B*' simply state that certain things have certain properties (namely those properties that things appear to have).

The standard response to a case of perspectival variance is this: the ways things appear to observers in a case of perspectival variance are *merely* ways things appear to observers in the relevant circumstances. They are not ways things

---

[1] I discuss Fine's conception of fragmentalism, as well as the way it differs from the conception proposed here, in [Lipman *forthcoming*].

genuinely are; they do not feature in a conception of the world as it is in itself [Williams 1979: 229]. This response typifies a whole family of views. Views differ in particular on the status they assign to the ways things appear to the observers. Are they perhaps not even properties that things appear to have (but then how to characterize the incompatibility between the observations?), or are they properties that we simply misattribute to things (but then whence this systematic error?), or are they properties that are somehow secondary or less-than-fully real yet such that we are somehow correct in attributing them to things (but then how to make sense of this metaphysical status hovering between the real and unreal?) For a discussion of these views, and their issues, see [Rosen 1994] and [Stroud 2000]. The finer-grained differences aren't relevant here. All that matters for our purposes is that any version of the standard response will at least deny, somehow, that things genuinely are the ways they appear to be to the observers.

The standard response cannot be avoided by saying that the apparent properties or relations merely turn out to have a higher adicity - that these cases simply reveal a hidden argument place. There is no such thing as 'inreasing the adicity' of a property whilst leaving it in place. We really just deny that the apparent *n*-adic property or relation is instantiated and replace it with a different *n+1*-adic relation (see [Boghossian 2006]). The latter is distinct from the former, not just because it has a different adicity, but also because the *n+1*-adic relation doesn't characterize the relevant way things appear to be to us. Sophie and Nora appear in incompatible ways. If it simply appeared to you *that Sophie is on the left of Nora relative to you*, and it appeared to me *that Sophie is on the right of Nora relative to me*, then there would be no incompatibility in the ways things appeared to us. But there *is* such an incompatibility and hence those three place relations aren't the relevant ways things appear to be to us. Sophie and Nora appear to have the two-place relations, and the standard response is to deny that these are instantiated.

The standard response is problematic due to the sheer range of properties that admit of perspectival variance. The perspectival variance of sensible properties is much-discussed. Imagine that we both observe a beetle. The back of a beetle is a so-called iridescent surface: it appears to have a different colour when observed under different angles. Say you see a greenish surface, I see a more blueish one. We have no good reason to privilege either of our perspectives; what would be the one true angle under which things show their true colours? Even if there was one true angle, there are many different ways in which such colour variance can arise – iridescence is merely one example (see [Cohen 2009: Ch.2]). And similar cases can be found for the way things sound, taste, in short for the typical secondary properties. Their discussion has a long history of course (see [Burnyeat 1979]), and they led many to a view according to which things cannot be genuinely coloured out there in the way that they appear to be in our experience (see the many views discussed by Stroud [2000]).

Many offer a version of the standard response in the case of sensible properties. But this isn't a stable resting point. There are far more radical cases of perspectival variance. Clear examples are found in phenomena treated by the special theory of relativity (see [Maudlin 2011: Ch.2], [Dainton 2010: Ch. 18] or – for a quick introduction – [Hawley 2009]). It was discovered that the velocity of

light is observed to be the same (roughly 300,000 kilometres per second) by any observer regardless of the velocity they observe themselves as having. Imagine that you're standing in the exact middle of a space shuttle and flip the switch of a laser that sends light to the front and back of the shuttle. From your perspective, the light has to travel the same distance to the front and back, and since it has a constant speed, it arrives at the same time at both ends. But now imagine I'm in a different space shuttle, and from my perspective your shuttle appears to be passing by. From my perspective, the light sent towards the front of your shuttle has to travel further (as the front moves away from the light), and since the light still has the same constant velocity, it arrives later than the light sent towards the back. Again there is no reason to privilege either one of our perspectives. We have here a case of perspectival variance involving a whole range of properties, including:

> *Constant velocities*: although the velocity of light is constant, things will appear to have different constant velocities to us (e.g. you seem to be at rest from your perspective but seem to be moving from mine).

> *Simultaneity*: certain events (those that are space-like separated, such as the light arriving at the front and back of the shuttle) will appear simultaneous from your perspective and non-simultaneous from my perspective.

> *Duration*: events will appear to have different durations to us.

> *Length*: things will appear to have different lengths to us.

> *Shape*: things will also appear to have different shapes to us.

> *Inertial mass*: things will appear to have a different inertial mass to us.

See [Sartori 1996: Ch.4] for the formal transformations of these properties across frames. Applying the standard response leads to a view according to which things don't really have the lengths, shapes, masses, or constant velocities they appear to have and events don't really have the durations or happen in the order they appear to.[2] Minkowski was the first to offer the standard response and famously pronounced that 'henceforth, space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality' [1908: 297].

Within a standard scientific image, many (indeed, virtually all) of the ways in which things appear to us are demoted to ways things *merely* appear to be. And yet we possess ample reason to think that these are ways things are in the world. The first of these is obvious: when confronted with an instance of perspectival

---

[2] Again: do not confuse such talk with talk of, for example, velocity-relative-to-a-frame; whatever that is, it's not the property that we observe things to have. If you just observed velocity-relative-to-one-frame and I observed velocity-relative-to-another-frame, what we observe would not be incompatible. But what we observe *is* incompatible. The relations to frames are not the relevant properties things appear to have to us.

variance, we have (be the definition of perspectival variance) no independent reason to believe the conditions of observation to be distortive. Observations are at least a guide to how things are in the world itself, and so in cases of perspectival variance we have prima facie reasons to believe that things are the ways they appear to us in our experience.

Along with observations, successful theorizing is another guide to what the world is like. And most of our very best theorizing about the world presumes that things have definite lengths, shapes and masses, and that all events occur in a definite temporal order. Examples are legion outside of physics: biology, geography, chemistry, psychology, economics, and so on, all arguably deal in objects of certain shapes and sizes interacting with each other in a patterned way over a definite course of time. We are however driven to a view on which there just aren't objects such as those presumed by any of these theories and there just isn't the temporal progression assumed by them over time. This is even in tension with successful theories found within physics itself: almost all interpretations of quantum mechanics, whether they be collapse based or non-collapse based interpretations, fail to be invariant across frames because of the role played by simultaneity in them (for clear discussions, see [Putnam 2005: 631] and [Maudlin 2011: Ch.7]).[3] Quantum mechanics is no less successful than the special theory of relativity, and yet it is naturally understood to predict incompatible results across different frames. Many of our most successful theories only apply to a world that is largely the way it manifests itself in our experience; and that again constitutes prima facie reason to believe that the world is the way these theories picture it to be, as including objects that instantiate properties that admit of perspectival variance.

If we stand back and look at the way our overall conception of the world is shaped by the standard response to cases of perspectival variance, we see a paradigm that is under mounting pressure. So the question arises: can we somehow make sense of things really being the incompatible ways they appear to be across perspectives?

3. Against metaphysical relativism

It's tempting to explore a type of metaphysical relativism at this point. Let metaphysical relativism be the view that the apparent facts in cases of perspectival variance do really obtain, but only relative to certain entities. We avoid saying that two incompatible facts obtain simpliciter: it's really the case relative to you *that Sophie is on the left of Nora*, and it's really the case relative to me *that Sophie is on the right of Nora*.[4] I don't have the space to discuss metaphysical relativism in the

---

[3] I say 'almost all' because Tumulka [2006] has proposed a version of the GWR spontaneous collapse theory – 'flashy GWR' – that reconciles relativistic space-time structure with the violation of Bell's inequality. For a critical discussion of this, see [Maudlin 2011: Ch.10].

[4] Distinguish metaphysical from semantic relativism, according to which the truth of an asserted or believed content is relative to contexts (as in [MacFarlane 2005]). The type of question that is of concern to us is whether Sophie can be both on the left and on the right of Nora, and this a metaphysical question about what objects can be like, and not a question about the ways in which

full detail it deserves. Let me however mention one simple objection raised by Moore [1997: Ch.3].[5]

Thus far we have a claim about facts, about *the fact that Sophie is on the left of Nora*, which is said to obtain relative to some object in the world, namely you. What we aim to capture however is the way *Sophie* and *Nora* appear to you from your perspective. How does to the relative obtaining of the fact bear on the involved objects? When a fact obtains simpliciter, the story is straightforward: for *the fact that Sophie sits* to obtain is plausibly just for Sophie to sit (or at least implies that this is so). As Moore notes, the obtaining of perspectival facts similarly needs to consist in the world's being a certain way [1997: 46]. The question is how the relative obtaining makes things be. What could it be for *the fact that Sophie is on the left of Nora* to obtain 'relative to you'? If we say that it's for Sophie to be on the left of Nora relative to you, then we have not captured the way Sophie appears to you. Again: if it appeared to you that Sophie is on the left of Nora relative to you, then it appeared to me that Sophie is on the right of Nora relative to me, and that isn't incompatible with the way it appeared to you. But Sophie and Nora *do* appear in incompatible ways.

If we say instead that for *the fact that Sophie is on the left of Nora* to obtain 'relative to you' is in some way or other for Sophie to be on the left of Nora, then the relativization becomes idle: by the same token, for *the fact that Sophie is on the right of Nora* to obtain relative to me would be for Sophie to be on the right of Nora, so that Sophie is both on the left and right of Nora after all. The whole point of the relativization was to avoid saying that incompatible facts obtain simpliciter.

Can a relativist refuse to explain how the relative obtaining of the fact bears on the way Sophie and Nora are? No: the aim is to admit the reality of what appears to you from your perspective, and what appears to you is that Sophie is on the right of Nora – so whatever story we offer, it needs to tells us how things stand with regards to *them*.

Although we need to consider possible responses and possible adaptations of the relativist view before we can conclusively reject it, I believe that Moore's argument reveals a serious conceptual instability. (See [Moore 1997: Ch. 3] for a defence of the argument against various responses, and see [Spencer forthcoming] for a recent defence of metaphysical relativism). There is a lesson to be drawn from the objection: in order to truly say that things are the way they appear to be from a certain perspective, it really needs to be the case that, for example, Sophie is both on the left and on the right of Nora. Though the talk about the relevant facts can be useful when speaking loosely, it ultimately has to give way to talk about the objects involved in those facts. What we need is not a framework that avoids the claim that *things* truly have incompatible properties; we need a framework that allows us to make sense of it.

---

representations of the world are true or false. There may be relations between the two kinds of relativisms, but such relations are relations between views that should be sharply distinguished.

[5] I have put the argument in my own terms. Lewis [1986: 204; 2002] raises a related worry for endurantists who hold that intrinsic properties are had relative to times. Fine [2005: §11] raises a related worry for what he calls 'external relativism' about tensed facts.

4. The possibility of a fragmented world

Why do we think that something cannot, for example, be both round and square? It's not a simple question of logical form. These facts have the form of '*a* is F' and '*a* is G' and there's nothing in these forms from which we can simply read their incompatibility. And yet we somehow see that something could not be both round and square.

Perhaps we think that something cannot be both round and square because we cannot imagine what such an object would be like. If we have an image of *a* as round and an image of *a* as square, we cannot combine these images into a single more detailed image of *a*. But now ask yourself this: why couldn't the two images just collectively depict the way the world is? Why is the inconceivability of a conjoined image evidence that at most one of the images can reflect what *a* is like, why couldn't the plurality of images as such be representative of the world? We seem to make the substantive assumption that the world is a unified place, so that the images need to be deemed squeezable into one before their collective contents can be taken to reflect what things are possibly like. But perhaps this unity assumption is mistaken; perhaps the world is not unified but fragmented.

To make sense of a fragmented world, I propose we distinguish between the obtaining of two facts and the co-obtaining of two facts. We don't normally distinguish these. We assume that when two facts each obtain, they *thereby* co-obtain. We need a concept of co-obtaining on which this isn't so, and such a concept isn't already explicit to us. What is it for two facts to obtain yet fail to co-obtain in this new sense? To answer this, we can start closer to home and think not of a fragmented world but of a fragmented body of information. As Lewis pointed out, it's plausible to think that our own beliefs form such a fragmented body of information:

> I used to think that Nassau Street ran roughly east-west; that the railroad nearby ran roughly north-south; and that the two were roughly parallel. [...] My system of beliefs was broken into (overlapping) fragments. Different fragments came into action in different situations, and the whole system of beliefs never manifested itself all at once. [...] I think the same goes for other corpora in which inconsistencies are successfully quarantined. The corpus is fragmented. Something about the way it is stored, or something about the way it is used, keeps it from appearing all at once. It appears now as one consistent corpus, now as another. The disagreements between the fragments that appear are the inconsistencies of the corpus taken as a whole. [Lewis 1982: 436].

I think that Lewis' description is plausible: two bits of information may each be believed, without being believed together.

It's not hard to see how there can be such fragmented bodies of information. Now think of the world as being just like such a fragmented body of information, think of it as a fragmented corpus of facts. Something about the nature of the world,

about the way the facts obtain, keeps the facts from obtaining all at once or altogether. The world appears now as one consistent corpus of facts, now as another. The disagreements between the fragments that appear are the inconsistencies of the world taken as a whole.

We have thus far talked about facts, instead of about the involved objects themselves. As we saw in the discussion of metaphysical relativism, our metaphysical object language should be one referring to the way the relevant objects are. Instead of attributing a co-obtainment relation to facts, we will express the co-obtaining of facts directly, using a sentential connective '∘' (cf. Fine's notion of coherence [2005: 281]). I propose we read, for instance, 'Sophie sits ∘ Nora sits' as 'Sophie sits *insofar as* Nora sits'. This reading is admittedly not ideal. In ordinary language 'insofar as' has a bare conjunctive reading but also an asymmetric reading on which the left side is in some sense explained by the right side - and it's only the conjunctive reading that fits (though certainly doesn't exhaust) the theoretical sense the notion is intended to have. From now on 'insofar as' is only used in the conjunctive sense.[6] I will also occasionally resort back to talk about facts, in the understanding that this can be translated into talk about the involved objects.

With the help of this newly introduced bit of ideology, we can explicitly state co-obtainment claims about objects. Ordinary conjunctive claims about objects don't imply the corresponding co-obtainment claims about those objects. We might say for example:

Sophie is on the left of Nora ∧ Sophie is on the right of Nora ∧ ¬(Sophie is on the left of Nora ∘ Sophie is on the right of Nora).

Sophie is on the left of Nora and Sophie on the right of Nora, but Sophie is not on the left of Nora *insofar as* Sophie is on the right of the Nora.

We noted that sometimes the incompatibility of facts isn't a matter of logic, as in the case of *a*'s being round and *a*'s being square. But sometimes it is, as in the case of *a*'s being round and *a*'s not being round. If we not only allow that *a* is round and that *a* is square, but also that *a* is round and not round, then we accept a metaphysics that requires revisions in our logic. I don't think that we need to take this route. We should distinguish between two kinds of conflicting facts:

'*A*' and '*B*' state *contrary* facts iff they cannot both obtain, i.e. iff necessarily ¬(*A*∧*B*).

'*A*' and '*B*' state *incompatible* facts iff they cannot co-obtain, i.e. iff necessarily ¬(*A*∘*B*).

Call pairs of sentences one of which is the negation of the other *contradicting* sentences. It seems clear that contradictory sentences always state contrary facts -

---

[6] Why not simply read 'Sophie sits ∘ Nora sits' as 'Sophie's sitting *co-obtains with* Nora's sitting'? One may do this of course but it requires we transform the embedded sentences into imperfect gerundial nominals, such as 'the tree's being leafless'. This makes for awkward sentences and leads away from straightforward talk about the involved object.

even for the fragmentalist: nothing can possibly be round and not round. Contradictory sentences stand in contrast to sentences such as 'a is round' and 'a is square', neither of which is the negation of the other, and yet which clearly express facts that conflict in some way. We adopt a fragmentalist view when we think that such sentences state incompatible facts, and not contrary facts, and hence that they can both obtain (though not co-obtain).

So all contradictory sentences state contrary facts. It's also natural to hold that – vice versa – all contrary facts are stated by contradictory sentences. But note that this isn't obligatory. If a fragmentalist believes that 'the box is full' and 'the box is empty' state contrary facts, this doesn't force the fragmentalist to also believe that one sentence expresses the negation of the other, that their logical will reveal the use of negation in one of them. She may simply be convinced that it's impossible that a box is both empty *and* full.

What about compatible facts, such as a's being round and a's being orange, do they necessarily co-obtain? I see no reason why they should. Think back to the plurality of images: one image may show a as round and green, and another may show a as square and orange, so that, according to the plurality of images, a's being round and a's being orange co-obtain as little as a's being round and a's being square. We can distinguish the possibility of compatible facts co-obtaining from the possibility in which they each obtain but fail to co-obtain, and this means that their co-obtaining is a contingent and substantive matter. A complete description of the world doesn't just need to capture everything that obtains, it needs to capture what co-obtains with what and which matters obtain yet fail to co-obtain.

There are then two ways in which modal space turns out to be richer than we normally assume it to be. First: for any two compatible facts, such as a's being round and a's being orange, it's possible that these co-obtain and it's possible that they fail to do so, i.e. we have it that $\lozenge(a$ is round $\circ$ a is orange$)$ and $\lozenge\neg(a$ is round $\circ$ a is orange$)$.[7] And, second: for any two incompatible facts, such as a's being round and a's being square, it's possible that they both obtain but impossible that they co-obtain, i.e. we have it that $\lozenge(a$ is round $\wedge$ a is square$)$ though $\neg\lozenge(a$ is round $\circ$ a is square$)$. When we try to imagine a round square, and fail, what we try to imagine is something that is round *insofar as* it is square. The fact that this is inconceivable, and indeed impossible, doesn't tell against it being possible that something is round *and* square without also being the one *insofar as* it's the other.

One may wonder whether a framework like this will turn out to be committed to the truth of contradictions and, if not, why not. After all when you observe Sophie and Nora, one might say that it doesn't *just* appear to you that Sophie is on the left of Nora but also that it appears to you that Sophie is *not* on the right of Nora. To answer this and related worries, we need to have a closer look at the inferential role of co-obtainment.

---

[7] The diamond '$\lozenge$' stands for metaphysical possibility, which behaves as it ordinarily does. Modal space simply includes fragmented possible worlds, i.e. worlds at which various co-obtainment claims are true.

5. A simple sentential semantics for co-obtainment

I will offer a simple sentential semantics of co-obtainment and ignore quantification and modality. The semantics goes in many ways back to the 'discussive logic' of Jaśkowski [1948] - with the important difference that Jaśkowski's discussive logic is paraconsistent, whereas the logic below isn't. (See also [Rescher and Brandom 1980], [Priest 2008] and, in particular, [Restall 1997]).

Let the set of sentences S consist of atomic sentences p, q, r, ... and be such that, if A and B are sentences, so are $\neg A$, $A \wedge B$, $A \vee B$ and $A \circ B$ (besides these, nothing else is in S). A model M is a pair $\langle W, v \rangle$ where W is a set of points and v is a function that assign either 1 or 0 to each of the atomic sentences relative to the points w in W.

The valuation v for the atomic sentences relative to points is extended to a valuation for all the sentences via the following recursive clauses (where w ranges over points in W):

$v_w(A \circ B) = 1$ iff $v_w(A) = 1$ and $v_w(B) = 1$,
$v_w(A \wedge B) = 1$ iff $v_w(A) = 1$ and $v_w(B) = 1$,
$v_w(A \vee B) = 1$ iff $v_w(A) = 1$ or $v_w(B) = 1$,
$v_w(\neg A) = 1$ iff $v_w(A) \neq 1$.

Truth in a model, $M \Vdash A$, is defined via the following recursive clauses (where p is an arbitrary atomic sentence):

$M \Vdash p$ iff $\exists w(v_w(p) = 1)$,
$M \Vdash A \circ B$ iff $\exists w(v_w(A \circ B) = 1)$,
$M \Vdash A \wedge B$ iff $M \Vdash A$ and $M \Vdash B$,
$M \Vdash A \vee B$ iff $M \Vdash A$ or $M \Vdash B$,
$M \Vdash \neg A$ iff $M \nVdash A$.

We define validity and logical truth as follows (where Σ is a set of sentences):

An argument from Σ to A is *valid*, written $\Sigma \vDash A$, iff, for every model M, if $M \Vdash \Sigma$ then $M \Vdash A$.[8]

A formula A is *logically true*, written $\vDash A$, iff, for every model M, $M \Vdash A$.

Note that the points correspond to the images of the toy models we discussed in the previous section.[9] The v-clauses say what sentences truthfully describe each point (or image, or fragment of world), and which sentences do not. There are v-clauses for conjunction, disjunction, negation and co-obtainment because the logic needs to handle the embedding of logically complex sentences in co-obtainment

---

[8] By $M \Vdash \Sigma$ we mean that $M \Vdash B$ for all $B \in \Sigma$.
[9] Alternatively, the points can be interpreted as the possible worlds known from standard modal logics, so that a single fragmented world (here represented by a single model) corresponds to a set of possible worlds in a frame of modal logic; cf. [Restall 1997].

sentences. For example, $A{\circ}(B{\circ}C)$ is true in a model if A and $B{\circ}C$ are true at a single point, and this requires that $B{\circ}C$ has itself a truth-value at points. The same applies to $A{\circ}\neg B$, $A{\circ}(B{\lor}C)$ or $A{\circ}(B{\land}C)$.

Let us run through the machinery stepwise. Start with the evaluations of atomic sentences relative to points and run the standard Boolean evaluations for complex sentences *relative to points*, treating co-obtainment as conjunction. The atomic sentences and co-obtainment sentences that are true at some point in W are all true in the model. And these truths are the basis for standard Boolean evaluations of negations, conjunctions and disjunctions in the model: taking negations of things that are false in the model to be true in the model, conjunctions of two truths to be true in the model, etc.

We can use the semantic machinery to see more clearly how the fragmentalist doesn't require a paraconsistent logic, and to see more clearly what formal features co-obtainment has. Let me start with the former.

A sentence is true *at a point* if and only if its negation isn't true *at that point*. This means that we never have a point where both a sentence and its negation are true. That is:

$$\vDash \neg(A{\circ}\neg A)$$

It cannot be the case that something obtains *insofar as* it doesn't obtain.

Similarly, any sentence in our language is *true in a model* if and only if its negation isn't true *in the model*. This means that the law of excluded middle and the law of non-contradiction hold:[10]

$$\vDash A{\lor}\neg A$$
$$\vDash \neg(A{\land}\neg A)$$

To illustrate an important consequences of this, consider a model where we have w_1 at which p is true but q isn't, and w_2 at which q is true but p isn't. As there are points at which p and q are true, they are true in the model. This means that $\neg q$ isn't true in the model. But given that $\neg q$ is true at w_1, $p{\circ}\neg q$ is true in the model. So $\neg q$ is false, yet true insofar as p is true (i.e. $\neg q$ is false but $p{\circ}\neg q$ is true). The fragmentation gives rise to negative sentences being true *insofar as* certain other things are true, even though they are false. You observe that Sophie is *not* on the right of Nora, but this is the case only *insofar as* Sophie is on the left of Nora.

It's important to note that this captures a natural understanding of negation in this context, reflecting how we think of local (or perspectival) absences versus global absences. Compare the way existence at a location and existence as such interact: an object exists when there is a location at which it exists, but it doesn't fail to exist when there is a location at which it doesn't exist. An object doesn't exist

---

[10] It can be easily checked that we also have:

$$B \vDash A{\lor}\neg A$$
$$A{\land}\neg A \vDash B$$

only when there is no location at which it exists. Whereas local existence suffices for global existence, local non-existence doesn't suffice for global non-existence. There is a natural asymmetry here. The same sort of asymmetry is found, in a fragmented world, between atomic facts and their negations. When an atomic fact obtains *insofar as* other facts obtain (or 'within a fragment'), this suffices for it to obtain simpliciter (or 'within the world as such'), but when the fact is absent *insofar as* other facts obtain (or absent 'within a fragment'), this does not suffice for the fact to be absent simpliciter (or absent from the world at large). Put more directly: if a is F *insofar as* b is G, this suffices for a to be F; but if a is not F *insofar as* b is G this does not suffice for a not to be F because it may still be F *insofar as*, say, c is H.[11] There is no reason why the fragmentalist should have a non-standard understanding of negation; and hence no reason why the fragmentalist requires a paraconsistent logic or admit true contradictions.[12]

Let me now turn to elucidating the formal properties and inferential role of co-obtainment. It can easily be seen from the semantics that co-obtainment is commutative and associative:

$$A{\circ}B \vDash B{\circ}A \text{ and } B{\circ}A \vDash A{\circ}B$$
$$A{\circ}(B{\circ}C) \vDash (A{\circ}B){\circ}C \text{ and } (A{\circ}B){\circ}C \vDash A{\circ}(B{\circ}C)$$

The commutativity of co-obtainment is underwritten by the fact that if A and B are true at some point, they aren't true in some particular order. To see why co-obtainment is associative, note that if we have $A{\circ}(B{\circ}C)$ this means that there is a point w where both A and $B{\circ}C$ are true (see the v-clause for $\circ$), and $B{\circ}C$ is only true at w if both B and C are true at w. This means that all three, A, B and C are true at w, which means that A${\circ}$B must be true at w together with C, and hence that $(A{\circ}B){\circ}C$ is true in the model.

Co-obtainment fails to be idempotent:

$$A{\circ}A \nvDash A$$
$$A \nvDash A{\circ}A$$

To see why we have $A{\circ}A \nvDash A$, consider a model where p is false at w_1 but true at w_2. In this model, $\neg p{\circ}\neg p$ is true given that there is a point where each is true (viz. w_1). And yet $\neg p$ is not true simpliciter, given that p is true at w_2. To see why we have $A \nvDash A{\circ}A$, consider a model where p is true but q is false at w_1 and where p is false but q is true at w_2. Here $p{\wedge}q$ is true in the model, but $(p{\wedge}q){\circ}(p{\wedge}q)$ isn't true, as there is no single point at which $p{\wedge}q$ is true.

Relatedly, we have the failure of simplifying and adjunctive rules for co-obtainment:

---

[11] Compare this to [Restall 1997] who takes an atomic sentence to be false in the model when false at a point, and consequently arrives at a paraconsistent logic (which, to be sure, is precisely what he aims for).

[12] To be sure, this is not to say that one could not develop the notion within a paraconsistent framework; it's just to say that there is nothing in the intuitive picture that requires or of itself leads to such a treatment.

$A, B \nvDash A \circ B$
$A \circ B \nvDash A$

For the failure of adjunction, consider a model where we have w_1 at which p is true and w_2 at which q is true. Here p is true and q is true because they are atomic sentences and there are points at which they are true. But $p \circ q$ isn't true, given that there is no point at which p and q are both true. The failure of adjunction is of course precisely what is required, and is in some sense the core feature of co-obtainment. For the failure of simplification, consider a model where p is true at w_1 and where q and $\neg p$ are true at w_2. Here $q \circ \neg p$ is true in the model, but $\neg p$ is not true in the model, given that p is true at w_1.

Co-obtainment is also non-transitive:

$A \circ B, B \circ C \nvDash A \circ C$

Consider a model where we have a point w_1 at which p and q are true but r isn't, and a point w_2 at which q and r are true but p isn't. In such a model, $p \circ q$ and $q \circ r$ are true, but $p \circ r$ isn't. Put informally, the failure of transitivity allows fragments to overlap.

Some of the failures of these types of inferences are solely due to the fact that we allow any kind of sentence to be embedded in co-obtainment sentences. For example, co-obtainment sentences that only embed atomic sentences are idempotent, and there is a restricted form of simplification for atomic sentences embedded in co-obtainment claims (where p is an atomic sentence):

$p \circ p \vDash p$ and $p \vDash p \circ p$
$A \circ p \vDash p$

For atomic idempotence, note that if one has a point at which p is true, one thereby has a point at which $p \circ p$ is true, and vice versa. For simplification note that if $A \circ p$ is true, then there is a point w at which A and p are true, but that suffices for p to be true in the model.

I take the above to include the most important principles for the purpose of solidifying our inferential grasp of the notion of co-obtainment. There are of course many more principles governing the co-obtainment notion; one can, for example, easily check that each of following hold:

$A \circ B \vDash A \circ A$
$A \circ (B \circ C) \vDash B \circ C$
$A \circ (B \wedge C) \nvDash B \wedge C$
$A \circ (B \wedge \neg B) \vDash C$
$A \circ (B \wedge C) \vDash (A \circ B) \wedge (A \circ C)$
$(A \circ B) \wedge (A \circ C) \nvDash A \circ (B \wedge C)$
$A \circ B \vDash A \circ (B \vee C)$
$A \circ (B \vee C) \vDash (A \circ B) \vee (A \circ C)$

$$(A \circ B) \lor (A \circ C) \vDash A \circ (B \lor C)$$

It cannot be stressed enough that the semantics draws out the inferential role of co-obtainment in ways we happen to find elucidating; the set-theoretic machinery is merely a heuristic tool to elucidate the logical structure of the co-obtainment notion. Sentences aren't true 'relative to' or 'at points' in our object language, the language which reflects how the fragmentalist understands the world. The points don't correspond to anything in the fragmentalist's ontology. Certain facts obtain *insofar as* other facts do, the rest is but metaphor and model.

6. Concluding remarks

It's normally thought that no case of perspectival variance can offer us reason to think that incompatible facts obtain. Evidence is always evidence that one amongst a range of possibilities obtains but the obtaining of incompatible facts – it's assumed – is never amongst the possibilities. According to the fragmentalist, the obtaining of incompatible facts *is* amongst the possibilities however and, hence, any single case of perspectival variance does offer reason to think that incompatible facts obtain. Since there are many such cases, the fragmentalist sees a strong inductive reason to think that the standard response is mistaken, and that the world is indeed fragmented in at least some cases of perspectival variance.

The fragmentalist framework sketched above does not constitute a theory of any particular perspectival phenomenon. There is however an obvious starting point for thinking of perspectival variance within the fragmentalist framework. Return to the case in which we observe Sophie and Nora. There are plausibly the following facts:

> Sophie is on the left of Nora ∧ Sophie is on the right of Nora ∧ (Sophie is on the right of Nora ○ Sophie is not on the left of Nora) ∧ (Sophie is on the left of Nora ○ Sophie is not on the right of Nora) ∧ (you experience Sophie and Nora ○ Sophie is on the left of Nora) ∧ your experience is veridical ∧ (your experience is veridical ○ Sophie is on the left of Nora) ∧ (I experience Sophie and Nora ○ Sophie is on the right of Nora) ∧ my experience is veridical ∧ (my experience is veridical ○ Sophie is on the right of Nora) ∧ (my experience is not veridical ○ Sophie is on the left of Nora) ∧ (your experience is not veridical ○ Sophie is on the right of Nora)

Sophie and Nora instantiate the two incompatible relations, though they don't instantiate the one *insofar as* they instantiates the other. When we change our orientation in space, our experience of them comes to co-obtain with different facts. The evaluation of the fragmentalist theories of perspectival phenomena is bound to depend on the explanatory value of these sorts of facts.

A crucial difference between forms of relativism and fragmentalism is that the latter doesn't refer to things that matters are relative *to*. The facts that together constitute a fragment of world cannot be identified in terms of what they are all

14

relative to. The fragments are anonymous. For example, in the interpretation of the special theory of relativity, the fragmentalist might think that the frames of reference give way to the facts that, the relativist would say, obtain 'relative to them'. For example the following facts might be said to co-obtain in our earlier case of the passing space shuttles:

> (you are at rest ○ the light reaches the front and back simultaneously ○ your arm has some length l ○ ...) ∧ (you move at a non-zero constant velocity ○ the light reaches the front before it reaches the back of the train ∘ the length of your arm is less than l ○ ...)

No reference is made to individual frames of reference. Properties are instantiated together in patterned ways: your being at rest goes together with things having certain constant velocities, lengths, shapes and masses whereas your moving at some velocity goes together with things having certain other velocities, lengths, shapes and masses. We have no means of identifying a perspective other than via the facts that appear to obtain from those perspectives.

The road ahead should be clear, as well as the reasons for venturing on it: fragmentalist theories safeguard the instantiation of properties that are manifest to us, and that we know to be of explanatory value. Just as there are many relativist views about different matters, there are bound to be many possible fragmentalist views out there in dialectical space.[13]

University of St Andrews and *University of Stirling*

References

Boghossian, P. 2006. What is Relativism?, in *Truth and Realism*, ed. P. Greenough and M. Lynch. Oxford: Clarendon Press: 13-37.

Burnyeat, M. 1979. Conflicting Appearances, *Proceedings of the British Academy* 65: 69-111.

Cohen, J. 2009. *The Red and the Real*. Oxford: Oxford University Press.

Dainton, B. 2010. *Time and Space, Second Edition*. Durham: Acumen.

Fine, K. 2005. *Modality and Tense: Philosophical Papers*. Oxford: Oxford University Press.

Hawley, K. 2009. Metaphysics and Relativity, in *Routledge Companion to Metaphysics*, ed. A. McGonical, R. Le Poidevin, P. Simons and R. Cameron. London: Routledge: 507-16.

Jaśkowski, S. 1948 (1969). Propositional Calculus for Contradictory Deductive Systems, *Studia Logica* 25: 143-157.

Lewis, D. 1982. Logic for Equivocators, *Noûs* 16/3:131-41.

Lewis, D. 1986. *On the Plurality of Worlds*. Oxford: Blackwell.

Lewis, D. 2002. Tensing the Copula, *Mind* 111/441: 1-14.

Lipman, M. A. *forthcoming*. On Fine's Fragmentalism, *Philosophical Studies*.

Macfarlane, J. 2005. Making Sense of Relative Truth, *Proceedings of the Aristotelian Society* 205/1: 305-323.

Maudlin, T. 2011. *Quantum Non-Locality and Relativity: Metaphysical Intimations of Modern Physics, Third Edition*. Oxford: Blackwell.

Minkowski, H. 1908 (1964). Space and Time, in *Problems of Space and Time*, ed. J. J. C. Smart. London: Macmillian: 297-312.

Moore, A. W. 1997. *Points of View*. Oxford: Oxford University Press.

Priest, G. 2008. Jaina Logic: A Contemporary Perspective, *History and Philosophy of Logic* 29/3: 263-78.

Putnam, H. 2005. A Philosopher Looks at Quantum Mechanics (Again), *British Journal of the Philosophy of Science* 56: 615-34.

Rescher, N. and R. Brandom 1980. *The Logic of Inconsistency: A Study in Non-Standard Possible-Worlds Semantics and Ontology*. Oxford: Basil Blackwell.

Restall, G. 1997. Ways Things Can't Be, *Notre Dame Journal of Formal Logic* 38/4: 583-97.

Rosen, G. 1994. Objectivity and Modern Idealism: What is the Question?, in *Philosophy in Mind: The Place of Philosophy in the Study of Mind*, ed. M. Michael, J. O'Leary-Hawthorne. Dordrecht: Kluwer Academic Publishers: 277-319.

Sartori, L. 1996. *Understanding Relativity: A Simplified Approach to Einstein's Theories*. Berkeley: University of California Press.

Sextus Empiricus. *Outlines of Scepticism*, ed. J. Annas and J. Barnes. Cambridge: Cambridge University Press.

Shoemaker, S. 2006. On the Ways Things Appear, in *Perceptual Experience*, ed. T. S. Gendler and J. Hawthorne. Oxford: Oxford University Press: 461-80.

Spencer, J. forthcoming. Relativity and Relationality, *Philosophy and Phenomenological Research*.

Stalnaker, R. 1984. *Inquiry*. Cambridge (Mass.): The MIT Press.

Stroud, B. G. 2000. *The Quest for Reality: Subjectivity and the Metaphysics of Colour*. Oxford: Oxford University Press.

Tumulka, R. 2006. A Relativistic Version of the Ghirardi–Rimini–Weber Model, *Journal of Statistical Physics* 125/4: 825–44.

Williams, B. 1979 (2005). *Descartes: The Project of Pure Enquiry*. Oxford: Routledge.